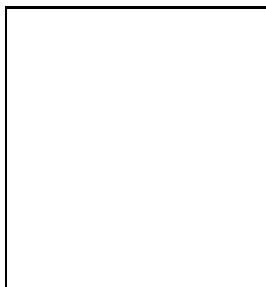


SN Ia DATA TO PROBE THE COSMOLOGICAL PRINCIPLE

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Recently collected SNIa data have been used to address the problem of measuring the cosmological parameters of the universe. Analysed in the framework of homogeneous models, they have yielded, as a primary result, a strictly positive cosmological constant. However, a straight reading of the published measurements, conducted with no a priori idea of which model would best describe our universe, at least up to redshifts $z=1$, does not exclude the possibility of ruling out the Cosmological Principle - and cosmological constant - hypotheses. It is here shown how the large scale (in)homogeneity of this part of the universe can be tested on our past light cone, using the magnitude-redshift relation, provided sufficiently accurate data from sources at redshifts approaching $z=1$ would be available.

1 Introduction

The widespread belief in large scale spatial homogeneity for our universe proceeds from an hypothesis brought to the status of Cosmological Principle. Its justification is based on two arguments: 1.The isotropy (or quasi-isotropy) of the CMBR around us. 2.The Copernican assumption that, as our location must not be special, this observed isotropy must be the same from any other point of the universe. However, of the two above arguments, only the first is observation grounded. The second, purely philosophical, cannot be directly verified.

Another common belief is that the inhomogeneities observed in the universe can be consistently smoothed out in an averaged homogeneous model. *This is absolutely wrong* and usually leads to improper uses of FLRW relations^{1,2}.

The discovery of high-redshift SNIa and their use as standard candles have resurrected interest in the magnitude-redshift (M-R) relation as a tool to measure the cosmological parameters of the universe. Data recently collected by two survey teams (the Supernova Cosmology Project

and the High- z Supernova Search Team), and analysed in the framework of homogeneous FLRW cosmological models, have yielded, as a primary result, a strictly positive cosmological constant, of order unity^{3,4}. If these results were to be confirmed, it would be necessary to explain how Λ is so small, yet non zero. Hence, a revolutionary impact in both cosmology and particle physics.

The purpose is here, assuming every source of potential bias or systematic uncertainties has been correctly taken into account in the data collecting, to probe the large scale homogeneity on our past light cone available with the SNIa measurements, thus testing the Cosmological Principle and cosmological constant hypotheses.

2 The magnitude-redshift relation to probe homogeneity

Consider any cosmological model for which the luminosity distance D_L is a function of the redshift z and of the parameters cp of the model. Assume that D_L is Taylor expandable near the observer, i.e. around $z = 0$,

$$D_L(z; cp) = \left(\frac{dD_L}{dz} \right)_{z=0} z + \frac{1}{2} \left(\frac{d^2 D_L}{dz^2} \right)_{z=0} z^2 + \frac{1}{6} \left(\frac{d^3 D_L}{dz^3} \right)_{z=0} z^3 + \frac{1}{24} \left(\frac{d^4 D_L}{dz^4} \right)_{z=0} z^4 + \mathcal{O}(z^5), \quad (1)$$

Therefore, the apparent bolometric magnitude m of a standard candle of absolute bolometric magnitude M is also a function of z and cp . In megaparsecs,

$$m = M + 5 \log D_L(z; cp) + 25 \quad (2)$$

Luminosity-distance measurements of such sources at increasing redshifts $z < 1$ thus yield values for the coefficients at increasing order in the above expansion. For cosmological models with high, or infinite, number of free parameters, the observations only produce constraints upon the parameter values near the observer. For cosmological models with few constant parameters, giving independent contributions to each coefficient in the expansion, the observed M-R relation provides a way to test the validity of the model, and, if valid, to evaluate its parameters.

For Friedmann models precisely, the expansion coefficients $D_L^{(i)}$ are independent functions of the three constant parameters H_0 , Ω_M and Ω_Λ , and can be derived from the expression of D_L ⁵. Therefore, accurate luminosity-distance measurements of three samples of same order redshift SNIa would yield values for $D_L^{(1)}$, $D_L^{(2)}$ and $D_L^{(3)}$ and thus select a triplet of numbers for the model parameters H_0 , Ω_M and Ω_Λ ^a.

1. If $\Omega_M < 0$, which leads to physical inconsistency - but cannot be excluded from the current data (see e.g. Fig. 6 of Riess *et al.*³, where the permitted ellipses can be extended to the $\Omega_M < 0$ region) - the homogeneity assumption have to be ruled out at this stage.

2. If $\Omega_M > 0$, the triplet can be used to provide a prediction for the value of the fourth order coefficient $D_L^{(4)}$. Now, if further observations at redshifts approaching unity are made, $D_L^{(4)}$ can be determined and compared to its predicted value, thus providing a test of the FLRW model.

^aIn fact, H_0 can be hidden in the magnitude zero-point $\mathcal{M} \equiv M - 5 \log H_0 + 25$. The SCP team calibrates \mathcal{M} from the data of a low-redshift sample and claims direct fitting of Ω_M and Ω_Λ . The HzSST calibrates M and claims, as a bonus, an estimate of H_0 .

If the ongoing surveys were to discover more distant sources, at redshifts higher than unity, the Taylor expansion would no longer be valid. Therefore, we should consider the fit of the Hubble diagram for accurately measured sources at every available scale of redshift⁵.

3 Simplified inhomogeneous models

Two statements remain to be proved at this stage: 1. The ruling out of the Cosmological Principle is not a purely academical possibility. Physically robust inhomogeneous models exist which can verify any observed M-R relation. 2. A non-zero cosmological constant is not mandatory, as $\Lambda = 0$ inhomogeneous models can mimic $\Lambda \neq 0$ Friedmann ones.

3.1 Example: LTB models with $\Lambda = 0$

Lemaître⁶-Tolman⁷-Bondi⁸ (LTB) models are spatially spherically symmetrical solutions of Einstein's equations with dust as the source of gravitationnal energy. They can thus be retained to roughly represent a quasi-isotropic universe in the matter dominated area. Indeed, a spherically symmetric model may be regarded as describing data that have been averaged over the whole sky, but not over distance².

The line-element, in comoving coordinates (r, θ, φ) and proper time t , is (with $c = 1$)

$$ds^2 = -dt^2 + S^2(r, t)dr^2 + R^2(r, t)(d\theta^2 + \sin^2\theta d\varphi^2). \quad (3)$$

Einstein's equations with $\Lambda = 0$ imply that the metric coefficients are functions of the time-like t and radial r coordinates, and of two independent functions of r , which play the role of the model parameters (cp). The radial luminosity distance is

$$D_L = (1 + z)^2 R. \quad (4)$$

The D_L expansion coefficients follow, as independent functions of the derivatives of the model parameters, evaluated at the observer ($z = 0$). These parameters, which are implicit functions of z , through the null geodesic equations, are present in each coefficient $D_L^{(i)}$ with derivatives up to the i th order⁵. LTB models are thus completely degenerate with respect to any M-R relation.

The LTB example has been chosen for calculation simplicity. The complete degeneracy obtained by relaxing only one symmetry of the corresponding homogeneous model, namely spatial translation, allows to infer that a less symmetric inhomogeneous model, where the spherical symmetry would for instance also be relaxed, would provide an even higher degree of degeneracy.

3.2 Illustration: the flat ($\Lambda = 0$) LTB model

Flat ($\Lambda = 0$) LTB solutions can be characterized by only one arbitrary function of r , namely $t_0(r)$, usually interpreted, for cosmological use, as a Big-Bang singularity surface. The $D_L^{(i)}$ coefficients can thus be expressed in term of the successive derivatives of $t_0(r)$, up to the i th order.

A comparison with the corresponding FLRW coefficients gives the following relations:

$$\Omega_M \longleftrightarrow 1 + 5 \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + \frac{29}{4} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} + \frac{5}{2} \frac{t_0'''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{1}{3}}}, \quad (5)$$

$$\Omega_\Lambda \longleftrightarrow -\frac{1}{2} \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + \frac{29}{8} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} + \frac{5}{4} \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{1}{3}}}. \quad (6)$$

This implies that a positive Λ in a FLRW interpretation of the data at $z < 1$ corresponds to a mere constraint on the model parameter in a flat LTB ($\Lambda = 0$) interpretation.

For their latest published results, the SCP team propose a FLRW interpretation, which they write as⁴ $0.8 \Omega_M - 0.6 \Omega_\Lambda \approx -0.2 \pm 0.1$. In a flat $\Lambda = 0$ LTB interpretation, this becomes^b

$$4.3 \frac{t'_0(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{1}{3}} t_p^{\frac{2}{3}}} + 3.625 \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{4}{3}}} + 1.25 \frac{t_0''(0)}{\left(\frac{9GM_0}{2}\right)^{\frac{2}{3}} t_p^{\frac{1}{3}}} \approx -1 \pm 0.1. \quad (7)$$

Note that, even with $\Lambda = 0$, degeneracy still remains, as an infinite number of $\{t'_0(0), t_0''(0)\}$ couples can verify Eq.(7).

4 Conclusions

Provided SNIa would be confirmed as good standard candles, data from this kind of sources at redshifts approaching unity could, in a near future, be used to test the homogeneity assumption on our past light cone. Would this assumption be discarded by the shape of the measured M-R relation, inhomogeneous solutions could provide good alternative models, as they are completely degenerate with respect to any of these relations, even with a vanishing cosmological constant. Would a FLRW type distance-redshift relation be observed, it would not be enough to strongly support the Cosmological Principle, as the possibility for an inhomogeneous model to mimic such a relation could not be excluded.

Therefore, at the current stage reached by the observations, a non-zero Λ is not mandatory, as $\Lambda = 0$ inhomogeneous models can mimic a $\Lambda \neq 0$ FLRW M-R relation.

In any case, to consolidate the robustness of future M-R tests, it would be worth confronting their results with the full range of available cosmological data, *analysed in a model independent way*.

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^b t_p and M_0 are not free parameters of the model. t_p is the time-like coordinate at the observer, and its value proceeds from the measured temperature at 2.73 K⁹. M_0 is a constant setting the scale of the comoving radial coordinate r .